Problem set 3

Answer Questions 10 and 12, and one of Questions 11 and 13.

Question 10:

Consider the case of combinatorial auctions with "two-minded" bidders: each bidder is interested in one of two possible sets S_i^1 and S_i^2 and values them, respectively, at v_i^1 and v_i^2. Show that the natural adaptation of the greedy auction shown in class for the single-minded case is no longer incentive compatible, in the following two variants:

1. Bidders are also interested in $S_i^1 \cup S_i^2$ and value it at $v_i^1 + v_i^2$.
2. Bidders are not interested in $S_i^1 \cup S_i^2$ and thus value it at $\max(v_1^1, v_1^2)$. (In this case the greedy auction will skip the lower ranked bundle of a player if he was already allocated the higher ranked bundle.)

Question 11 [Mechanism design without money]:

Given a metric space $X = \mathbb{R}$ with the usual Euclidian distance, let $d(x, y)$ denote the distance between points $x, y \in \mathbb{R}$. Every agent i of n agents has a location $x_i \in \mathbb{R}$, and a facility should be located at some point $y \in \mathbb{R}$. The cost of an agent i is the distance between his location and the location of the facility; i.e., $cost_i(x_i, y) = d(x_i, y)$. Define the *maximum cost* of a location y with respect to a location profile $x = (x_1, \dots, x_n) \in \mathbb{R}^n$ as the maximum distance from y to any point x_i (for one of the agents).

1. Show a truthful mechanism (without money) that has an approximation ratio of 2 for the maximum cost (i.e., that always returns a solution that is within 2-factor of the optimal solution), and prove that this is tight. (i.e., any deterministic truthful mechanism has an approximation ratio of at least 2 in the worst case).
2. A randomized mechanism is a function from $\mathbb{R}^n$ to probability distributions over $\mathbb{R}$. The distance of agent i is defined naturally as the expected distance given the distribution. Show a randomized and truthful social choice function that has an approximation ratio of 3/2 for the maximum cost.

Question 12 [Second-price auction]:

1. Prove that for every false bid $b_i \neq v_i$ by a bidder in a second-price auction, there exist bids b_{-i} by the other bidders such that i's utility when bidding b_i is strictly less than when bidding v_i.
2. Consider a second price auction with n bidders and suppose a subset S of the bidders decide to collude, meaning that they submit false bids in a coordinated way to maximize the sum of their payoffs. Establish necessary and sufficient conditions on the set S (in terms of the private valuations of the bidders) such that the bidders of S can increase their collective payoff via non-truthful bidding.
3. Prove that for every single-parameter problem, every implementable allocation rule is monotone.

Question 13 [Combinatorial auctions with subadditive bidders]:

Consider a combinatorial auction with a set M of m goods and n bidders. Assume that the valuation function of every bidder $v_i(\cdot)$ is normalized, monotone and *subadditive* (i.e., for every disjoint sets T_1, T_2, $v_i(T_1) + v_i(T_2) \geq v_i(T_1 \cup T_2)$).

Consider the winner determination problem, and for now, ignore payments and truthfulness, rather consider only poly-time social welfare maximization. Given M and $v_1, \ldots, v_n$, call the winner determination problem *lopsided* if there is an optimal allocation of goods in which at least half of the total SW of the allocation is due to players that were allocated a bundle with at least $\sqrt{m}$ goods. (i.e., if $\sum_{i \in A} v_i(T_i^*) \geq \frac{1}{2} \sum_{i=1}^{n} v_i(T_i^*)$, where T^* is the optimal allocation and A is the subset of bidders i with $|T_i^*| \geq \sqrt{m}$.)

1. Show that in a lopsided problem, there is an allocation that gives all the goods to a single player and achieves an $\Omega(1/\sqrt{m})$ fraction of the maximum-possible SW.
2. Show that in a problem that is not lopsided, there is an allocation that gives at most one good to each player and achieves an $\Omega(1/\sqrt{m})$ fraction of the maximum-possible SW. [hint: use subadditivity.]
3. Give a poly-time $O(\sqrt{m})$-approximate winner determination algorithm for subadditive valuations. [hint: make use of a graph matching algorithm].
4. Give a poly-time, $O(\sqrt{m})$-approximate, truthful combinatorial auction for subadditive valuations.